

# Russell on Modality

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It is well known that Bertrand Russell did not champion modal logic, though this is not to say that he was completely silent on the subject of modality. Rather, there are occasional passages in his work where he discusses the alethic notions of necessity, possibility and impossibility with a preference towards eliminative logical analyses of these notions. It is these efforts which will be the focus of this paper. In particular, my main points of reference will be his posthumously published landmark 1905 *Necessity and Possibility* as well as some small portions of his *The Philosophy of Logical Atomism*. Although more than a decade apart, the basis of what Russell had to say on modality in these writings is common between them. Unsurprisingly, very little has been written on the subject of Russell and modality. In light of the fine 20th century developments in modal logic, Russell's idiosyncratic and skeptical views about modal notions seem archaic and just wrong. My simple aim is primarily to investigate what little Russell had to say about modality for the sake of historical interest and Russell scholarship. Byproducts of this investigation are a look at alethic notions as particularly expressed in ordinary language and a background advocacy of modal logic.

In *The Principles of Mathematics* ([8], p. 454.) Russell adopted a graded notion of necessity from G.E. Moore [4], but this was of very little help to him and he wasn't long satisfied with it.([2], p. 121.)

Russell's main eliminative treatment of modality, which attributes necessity as a predicate of propositional functions rather than propositions, first appears in *Necessity and Possibility*:

“The propositional function ‘ $x$  has the property  $\phi$ ’ is necessary if it holds of everything; it is *necessary throughout the class of  $u$*  if it holds of every member of  $u$ .”

“The propositional function ‘ $x$  has the property  $\phi$ ’ is *possible* if it holds of something; if it is *possible within the class  $u$*  if it holds of some member of  $u$ .” ([6], p. 518.)

So, one may call a propositional function:

- *necessary*, when it is always true
- *possible*, when it is sometimes true
- *impossible*, when it is never true

For example, suppose a ball  $x$  is randomly drawn from a bag which contains a number of balls. If all the balls in the bag are red, then the propositional function ‘ $x$  is red’ is necessary, if some of the balls are red, then the propositional function is possible and if none of the balls are red then the propositional function is impossible. This is a plausible, if somewhat peculiar account of the use of modal terms and the

connections between necessity and universal quantification and possibility and existential quantification are apparent. However, it can be no more than a modest account by contemporary standards if modal terms can not be meaningfully attached to propositions.

But Russell was averse to the idea of propositional modality as a primary subject for logic, one reason being that there is no one fundamental logical notion of necessity, nor consequently of possibility. Despite the fact that modal terms are naturally attached to propositions, there is nothing fundamental in these notions of modality for which there is a place in pure logic. As far as logic should be concerned, propositions are simply true or false and modal distinctions are extraneous to the purpose of logic as he sees it. The following quote goes some way towards illustrating Russell's mindset.

The traditional view was that, among true propositions, some were necessary, while others were merely contingent or assertoric; while among false propositions some were impossible, namely, those whose contradictories were necessary, while others merely happened not to be true. In fact, however, there was never any clear account of what was added to truth by the conception of necessity. ([5], p. 165.)

The gist here is that propositions are simply true or false and that logic should not be concerned with setting forth rules of inferences for modal propositions. With propositional functions however, attributions of necessity, possibility and impossibility can be used to tell us something about a logical property of the propositional function in question, namely something about its satisfiability.

After considering several definitions of necessity, Russell does in *Necessity and Possibility* offer a certain secondary reductional definition of propositional necessity based on his system of necessity as a predicate of propositional functions. In building up to his primary definition, Russell writes "it is possible to regard a proposition as *necessary* when it is an *instance* of a type of propositions all of which are true". ([6], p. 517.) In other words, a proposition is necessary if it is an instantiation of a propositional function which is necessary.

For example, the proposition 'Russell is either a man or not a man' may be called necessary because the statement remains true if we substitute anything else in place of Russell. Similarly, the proposition 'If Russell is a man, he is a mortal' remains true if we put anything else in place of Russell, and may therefore be called necessary. In this way, the necessities of logical tautologies, analytic truths and empirically necessary truths can be accommodated.

Russell points out that there are propositions which do not seem to quite fit this approach:

For example, we feel certain of the truth of all propositions of the type: " $x$  either is not a moment of time, or is a moment of time subsequent to the death of Cromwell, or is a moment of time preceding the Restoration"; yet we should hesitate to call propositions of this type *necessary*.

For we realise at once that the truth of all propositions of this type is a deduction from “the death of Cromwell preceded the Restoration”, which must be a contingent proposition if any proposition is to be contingent. Yet perhaps this feeling could be turned into its opposite. For if anybody said “Such and such an event happened before the death of Cromwell but after the Restoration”, we should reply “that is *impossible*, because Cromwell died before the Restoration”.([6], p. 519.)

This example exhibits a bad confusion of modal notions which ironically is the sort of thing that modal logic could be used to deal with. Suffice it to say, this purported counterexample I think falls short of hitting the mark simply because accompanying any necessity of this proposition is the implicit qualification ‘given that the death of Cromwell preceded the Restoration’. So it is the proposition ‘If the death of Cromwell preceded the restoration, then  $x$  either is not a moment of time, or is a moment of time subsequent to the death of Cromwell, or is a moment of time preceding the Restoration’, which is actually necessary in this sense.

Whilst this definition of propositional necessity is reasonable, a correlative definition of possibility is less persuasive. Russell offers some brief comments regarding a correlative definition of propositional possibility which are rather vague and a little confounding. Rather than trying to render them intelligible, I leave it to the reader to make what they will of these comments:

“A proposition in which (say) Socrates occurs is *possible* if there is something which can be substituted for Socrates so as to make the proposition true (this is intended to cover the case where the proposition in which Socrates occurs is itself true). The latter definition leads to more or less paradoxical consequences, for example, that Socrates might be a triangle, because there are triangles. These consequences, however, are only paradoxical because we know that Socrates is a man; and the proposition “Socrates is a man and a triangle” is impossible according to the definition.([6], p. 517.)<sup>1</sup>

Defining propositional modality in such a way requires some care for if an operative constituent in a proposition occurs two or more times, the proposition will be an instance of three or more propositional functions. For example, ‘Russell is identical with Russell’ is necessary when regarded as an instance of ‘ $x$

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<sup>1</sup>Russell continues “If we allow ourselves to take account of the actual properties of Socrates in estimating whether a proposition in which he occurs is possible or not, there is no ground for stopping short of *all* his properties; and then true propositions in which he occurs are possible and false propositions in which he occurs are impossible. I think, however, that something like this theory covers a good many cases in which possibility is commonly affirmed. Suppose I take a cab, and its number has five figures; I shall feel that it might have had four figures. In this case, all that is meant seems to be: “This is a London cab, and some London cabs have numbers consisting of four figures.” In such cases, the subject of the proposition is felt as a variable: it is not fully determinate, but as an indefinite member of some class.”

is identical with  $x$ ' but is only possible when regarded as an instance of ' $x$  is identical with Russell'. This ambiguity can be avoided by adding a qualification. A proposition of which say 'Russell' is a constituent is to be called necessary with respect to 'Russell' if the propositional function obtained by removing all instances of 'Russell' is necessary. Thus the proposition 'Russell is identical with Russell' is necessary with respect to 'Russell'. We can even consider this modality with respect to logical constants. For example the proposition 'Russell is identical with Russell' is possible with respect to identity, because 'Russell has the relation  $R$  to Russell' is possible.

So Russell here may be said to have a modal theory of sorts, providing semantic accounts of modal terms. Though there is certainly no trace of a modal logic nor can it really be considered that there is a reductive interpretation of what could commonly count as a modal logic. To start with, there is no indication of a means with which to carry out a systematic study of valid forms of alethic modal inference. His conceptions and definitions are quite peculiar and are at odds with several defining and essential aspects of common modal logic. With his system, a propositional function is not possible unless at least one actual argument satisfies it. But consider the propositional function ' $x$  is a unicorn'. Since nothing actually is a unicorn, ' $x$  is a unicorn' is an impossible function for Russell. But in a way, instantiations of this propositional function are logically contingent propositions. Another peculiarity is that there is a primary division between those things which are true or false and those things which are necessary, possible or impossible. Propositions are either true or false, but should not be primarily predicated as necessary, possible or impossible. On the other hand, propositional functions are neither true nor false, but can be primarily predicated as necessary, possible or impossible. To say that something is necessary but not true highlights the strange sense in which Russell borrows these modal terms and applies them to propositional functions. However, given Russell's secondary, eliminative definition of propositional modality in terms of propositional function modality, the two fundamental modal inferences referred to as the *Rule of Possibility* (whatever is true is possibly true) and the *Rule of Necessity* (whatever is necessarily true is true) to a certain extent hold with regard to propositions. Another aspect of modal logic concerns the iteration of modalities, and it is not clear how there could be any such thing in Russell's account.

I now turn to a subsequent Russellian account of propositional modality to be found in *The Philosophy of Logical Atomism*. At the beginning of lecture seven, Russell is concerned with clarifying his use of the term 'possible' and sketches an eliminative analysis of this modal term as it is attached in ordinary expression to propositions. As established, a propositional function is possible according to Russell when there is some instance of it which is true. Here he contends that ordinary uses of the word 'possible' with regard to propositions are derived from his notion by a process. To begin with, he considers what is meant by the proposition:

It is possible that it may rain tomorrow (1)

According to Russell, “there is in this proposition some constituent which, if you turn it into a variable, will give a propositional function which is sometimes true.” ([7], p. 222.)<sup>2</sup> So basically, the proposition ‘It will rain tomorrow’ contains a constituent, ‘tomorrow’, and replacing this constituent with a variable,  $d$ , results in the propositional function

It will rain on  $d$  (2)

where  $d$  ranges over a set of days. In other words, ‘It will rain tomorrow’ is just an instance of the propositional function ‘It will rain on  $d$ ’.

Now, we don’t know whether or not it will rain tomorrow, but we do know that the propositional function (2) is possible. This logical analysis is important, because Russell holds that (1) can be reduced to the conjunction of the following two statements:

It is known that ‘It rains on  $d$ ’ is true for some value/s of  $d$ . (3)

and

It is not known whether or not ‘It rains on  $d$ ’ is true when  $d$  has the value ‘tomorrow’. (4)

The first ensures that we know that the propositional function is not impossible. If it were impossible then it would be false for all values of  $d$ , including ‘tomorrow’. If we knew that the propositional function was impossible we would know that ‘It will rain tomorrow’ is not true, and obviously this is not known. Having asserted in (3) that the relevant propositional function is possible we then assert in the (4) that we don’t know whether the propositional function is true or false when the variable takes the value, ‘tomorrow’.

Russell also considers what would be meant by the statement

Unicorns are possible (5)

Things get a little confusing here. When Russell says, for instance, that ‘lions exist’, he does not mean the same thing as when he says ‘lions are possible’. When he says that ‘lions exist’, that means that the propositional function ‘ $x$  is a lion’ is a *possible* one, in his technical sense of the word possible as it relates to propositional functions. The proposition ‘lions are possible’ for Russell is a different sort of statement, and essentially comes down to the same thing as the analysis for (1); Russell contends that “as to what you would mean by saying that unicorns are possible, it would always come down to the same thing as “It is possible it may rain tomorrow.”” ([7], p. 223.)

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<sup>2</sup>Russell also here mentions the further qualification I discussed earlier: “You ought not therefore to say of a proposition simply that it is possible, but rather that it is possible in respect of such-and-such a constituent. That would be a more full expression”

So in saying that unicorns are possible in this sense, we are saying that the proposition ‘there are unicorns’ is an instance of the propositional function ‘there are  $x$ ’, which we know to be sometimes true but which we do not know to be true when  $x$  takes the value ‘unicorns’. This gives us the following:

It is known that ‘there are  $x$ ’ is possible, that is, true for at least one value of  $x$  (6)

It is not known whether or not ‘there are  $x$ ’ is true when  $x$  takes the value ‘unicorns’ (7)

In this way Russell analyses ordinary usage of ‘possible’ in terms of his own particular use of this modal term. However, this logical analysis does not work in all cases where we may conceivably attach ‘possible’ to propositions.

Take the statement,

Italy are the 2006 World Cup winners (8)

Since (8) is true, that Italy are the 2006 World Cup winners is possible; whatever is true is possible<sup>3</sup>. Furthermore, it is known that Italy are the 2006 World Cup winners. Yet according to this Russellian account, the possibility of (8) would be reduced to the following two statements:

It known that ‘ $x$  are the 2006 World Cup winners’ is true for some value of  $x$  (9)

and

It is not known whether or not ‘ $x$  are the 2006 World Cup winners’ is true when  $x$  takes the value ‘Italy’. (10)

Whilst (9) is certainly the case, (10) is certainly not.

Similarly with the statement,

lions are possible (11)

which is true because lions actually exist; whatever actually exists possibly exists. Yet according to Russell’s account (11) implies the following:

It is not known whether or not ‘there are  $x$ ’ is true when  $x$  takes the value ‘lions’. (12)

which is clearly not the case. So at this stage it is apparent that Russell’s definition of possibility here is limited.

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<sup>3</sup> $p \supset \diamond p$

Although generally dismissive of Russell’s account, I think that it can plausibly be used to explain away his first example concerning the possibility of whether or not it will rain tomorrow. He has made a mistake however in using his second example concerning the possibility of unicorns. Usually, when someone utters ‘unicorns are possible’, they are doing anything but implying that

It is not known whether or not ‘there are  $x$ ’ is true when  $x$  takes the value ‘unicorns’

Their employment of the term ‘possible’ has nothing to do with what they do not know; there is no implication that knowledge is unavailable. Rather, they *do* know that ‘there are  $x$ ’ is *not* true when  $x$  takes the value ‘unicorns’. Given this and Russell’s account, in this sense unicorns are impossible. Yet in another sense one might still want to meaningfully say that unicorns are possible. These apparently inconsistent interpretations signal for the introduction of a disambiguation.

In the sense indicated by Russell’s passage, the statement ‘unicorns are possible’ means something like

It is possible that unicorns exist (13)

If someone genuinely does not know whether or not unicorns exist, then they may genuinely assent to (13). We may assume though that most people do know that unicorns are mythical and do not actually exist, therefore they have something else in mind if they assent to ‘unicorns are possible’. In this sense, the statement ‘unicorns are possible’ means something like

The existence of unicorns is possible (14)

Whereas the former is false the latter may be considered true. If we know that something is not the case, then it is not possible that that something is the case. But in some sense it may still be possible<sup>4</sup>. So there seems to be two senses of the word possible as ordinarily expressed; I will return to this point later once I have more thoroughly established this distinction.

Another question which naturally arises is that given this eliminative definition of possibility, are there ways to derive the correlative notions of ‘necessity’ and ‘impossibility’ and what would be their logical connections? How would Russell have explained away the ordinary attachments of ‘necessary’ and ‘impossible’ to propositions, such as

That it will rain tomorrow is necessary (15)

or

That it will rain tomorrow is impossible (16)

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<sup>4</sup> $\nexists \neg p \supset \neg \diamond p$

As discussed, Russell reduces the statement

It is possible that it may rain tomorrow (17)

to the following pair

It is known that 'It rains on  $d$ ' is true for some value/s of  $d$ . (18)

and

It is not known whether or not 'It rains on  $d$ ' is true when  $d$  has the value 'tomorrow'. (19)

so the obvious approach would be to modify either (18) or (19)

The following modification of (19) will suffice I think to serve as a reduction of (16).

It is known that 'It rains on  $d$ ' is not true when  $d$  takes the value 'tomorrow'. (20)

As for a reduction of (15), one option is the following modification of (18)

It is known that 'It rains on  $d$ ' is true for every value of  $d$ . (21)

which would give us a necessary propositional function hence a necessary proposition under Russell's account of propositional necessity being derived from propositional function necessity. Despite this, the situation proposed by (21) is not the case and this strategy is unrealistic and unacceptable.

Alternatively, we could try to make a modification to (19) such as

It is known that 'It rains on  $d$ ' is true when  $d$  takes the value 'tomorrow'. (22)

This strategy, although realistic, does not seem quite right as a definition of necessity. Just because we are certain that it will rain tomorrow does not mean that it is necessary that it will rain tomorrow.

So whilst a reasonable definition of a correlative notion of impossibility is available, how to define a correlative notion of necessity in these terms, let alone logically link it with possibility fails, or is at least far from clear. In fact, I believe such attempts are misguided for as will be explicated the type of possibility which Russell's discussion can at best account for does not have necessity as a correlative notion. What is becoming clear though is that in general, possibility, together with impossibility and necessity, are logical notions which cannot be analysed in terms of what people know or believe.

The following account of alethic modality provides a basis from which we can better assess Russell's conception of propositional possibility and pinpoint its inadequacies. Girle [1] considers the application of

modal logic to the alethic notions as they are expressed in ordinary language. The starting point of his consideration is the suggestion that there are two types of possibility embedded in ordinary English, firstly a qualifiable possibility expressed by the phrase ‘possible for’ and secondly a variable possibility expressed by the phrase ‘possible that’.

The first type of possibility, ‘possible for’, can be qualified by words such as logically, financially, legally, physically and metaphysically. Before qualification we have statements such as:

- It is possible for pigs to fly
- It is possible for Bob to run

When qualified we have:

- It is *logically* possible for pigs to fly
- It is *physically* possible for Bob to run

The corresponding term that goes with qualifiable possibility is ‘necessity’, which gives us statements such as

- It is necessary for Jim to sell his house
- It is *financially* necessary for Jim to sell his house

We can also see that logical equivalences between the two terms which hold in normal modal logics make sense. For example,

- It is necessary for Jim to sell his house iff it is not possible for Jim not to sell his house

The second type of possibility, ‘possible that’, can be varied by terms such as slightly, hardly, barely, greatly, faintly, quite, and vaguely. Before variation we have statements such as:

- It is possible that Sally is cooking.

When varied we have:

- It is quite possible that Sally is cooking.

The corresponding term that goes with variable ‘possibility’ is ‘definite’. For example,

- It is definite that Sally is cooking.

or

- It is quite definite that Sally is cooking.<sup>5</sup>

The following links ‘possible’ and ‘definite’:

- It is definite that Sally is cooking iff it is not possible that Sally is not cooking.

There are of course logical connections between variable and qualifiable possibility, exemplified by the following entailments. Whilst the following holds

- If it is possible that Bob is buying a boat then it is possible for Bob to be buying a boat.

the converse entailment does not hold. Two other entailments which hold are:

- If it is not possible for Bob to be buying a boat then it is not possible that Bob is buying a boat
- If it is necessary for Bob to be in the room then it is definite that Bob is in the room

As is often the case with informal, natural language statements, the validity of some of these entailments may be vulnerable to certain interpretations given certain contexts and qualifications, and this distinction may be at times confused, but the general idea of this distinction is no doubt a correct one. This outline will suffice for my purposes, though no doubt these ideas are open to further investigation.

Now, with this distinction established, we are in a better position to scrutinize Russell’s examples. In terms of this distinction the following two statements:

it is possible for it to rain tomorrow

it is possible but not definite that it will not rain tomorrow

entail the following statement

it is possible that it may rain tomorrow

It is by now apparent that Russell’s eliminative treatment of ‘possibility’ as attached to propositions at best deals with the ‘possible that’ type of possibility. The correlative notion of this variable possibility is *definiteness*, not necessity. So he is here really roughly equating a proposition’s possibility with uncertainty as to whether the state of affairs affirmed by the proposition has come about (or did or will come about). His account has nothing to do with a general concept of possibility as contingency and its correlative concept of necessity. It is confined to the ‘dictionary’ meaning of a possible thing, as something which may be true or may be the case, as something concerning which one has no knowledge to the contrary. This is the type

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<sup>5</sup>It is apparent that qualifying ‘definite’ can tend to diminish its force

of possibility we associate with probability, evidential support and future indeterminacy, accompanied by terms which express degrees of affirmation such as vaguely, hardly, barely, quite and certainly. Although taken from *Necessity and Possibility*, the following quote is illustrative of this particular conception of possibility and Russell's mindset:

The subject of *probability* is one which is naturally associated with modality: the probability of a proposition's being true may be supposed to be [a] measure of its greater or less degree of possibility. ([6], p. 519.)<sup>6</sup>

Russell's account thus can to an extent satisfactorily explain away variable possibility as expressed in ordinary English. On the other hand, Russell's hasty treatment of possibility leaves behind no means with which to analyse the notions of qualifiable possibility and necessity, and furthermore, there is no way to analyse the logical connections between variable and qualifiable possibility. A correlation of qualifiable possibility and necessity with propositional function possibility and necessity comes to mind but would be inappropriate.

Referring back to Russell's examples, whereas the type of possibility present in the statement 'It is possible that it may rain tomorrow' is variable possibility, the type of possibility present in the statement 'unicorns are possible' is commonly taken to be qualifiable possibility. As to what one would mean by saying 'unicorns are possible', a qualification of this statement would be

It is physically possible for unicorns to exist

in the sense that say the evolution of a horned horse is physically possible, or

It is logically possible for unicorns to exist

With this distinction at hand, we can make sense of modal statements involving multiple instances of the term 'possible' which would otherwise be nonsensical within Russell's framework. For example, 'although it is surely not possible that unicorns exist, it is possible for unicorns to exist'; 'Although it is definite that pigs do not fly, it is logically possible for pigs to fly'; 'Although since Bob is buying a Ferrari it is not possible that Bob is buying a Porsche, it is financially possible for Bob to buy a Porsche'. So we must keep in mind that these two concepts of possibility are related but separate. Indeed, any approach to studying modality which can accommodate both conceptions of possibility is better than any approach which is limited to one. All this can be used to support the idea that modal logic will give us a clearer account of possibility and necessity/definiteness than can be given by providing an eliminative

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<sup>6</sup>Incidentally, Russell continues "Thus it would be necessary, in order to show that modal distinctions are never required, to produce a theory of probability in which no such distinctions are invoked." In a way, development of the alternative probabilistic semantics for modal logic has somewhat validated this inadvertently prognosticative claim.

logical analysis of these notions *à la* Russell.<sup>7</sup>

Given Russell’s motivations and philosophical stances, his lack of a modal logic and his modest ideas on modality come as no surprise. Whilst he occasionally informally indulged in rich modal talk<sup>8</sup>, he was neither a modal logician nor a serious modal theorist. It is fair to say that the subject of modality is one area where his characteristic approach of reductive logical analysis, which he so successfully applied on many other occasions, was not so successful. In *Necessity and Possibility*, Russell introduces his idea of attaching necessity and possibility to propositional functions rather than propositions, thus analysing away modal expressions with quantificational expressions. Whilst his particular use of these alethic modal terms is plausible, arguing against propositional modality as a central logical concept is unwarranted and statements such as “the whole doctrine of modality only applies to propositional functions, not to propositions” ([7], p. 204) are egregious. Russell’s treatment of the notion of possibility in *The Philosophy of Logical Atomism* is idiosyncratic and limitedly deals with one ordinary sense of possibility. From one perspective it can be said that he here eliminated an alethic modal notion only to introduce epistemic modal notions, which themselves are not unproblematic. A definitive account of possibility will need more than just a reduction to the satisfaction of several conditions, some of which are epistemic.

Russell concludes *Necessity and Possibility* by saying that

so far as appears, there is no one fundamental logical notion of *necessity*, nor consequently of *possibility* ... [thus] the subject of modality ought to be banished from logic, since propositions are simply true or false, and there is no such comparative and superlative of truth as is implied by the notions of contingency and necessity.

Of course, this line of argument will do little to derail the validity of modal logic as we know it. If anything, the variety of notions of necessity (and, correlatively, possibility and impossibility) should, and has served to drive the development of a general framework for the corresponding variety of appropriate modal logics.

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<sup>7</sup>Garle outlines a multiply modal logic to deal with these notions, where  $\diamond p$  translates to ‘possible for p’,  $\Box p$  translates to ‘necessary for p’,  $\mathbf{M}p$  translates to ‘It is possible that p’ and  $\mathbf{D}p$  translates to ‘It is definite that p’. Two entailments that should be valid are  $\mathbf{M}p \supset \diamond p$  and  $\Box p \supset \mathbf{D}p$

<sup>8</sup>For example, “mathematics takes us ... into the region of absolute necessity, to which not only the actual world, but every possible world, must conform” ([10], p. 55.)

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